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Higgs ξ-inflation for the 125–126 GeV Higgs: a two-loop analysis

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ABSTRACT: A non-minimal coupling ξ of the Standard Model Higgs field to gravity can give rise to inflation, but large ξ is required and thus leads to a violation of perturbative unitarity at $M_{\rm Pl}/\xi$, which is well below the inflationary scale $M_{\rm Pl}/\sqrt{\xi}$. We re-examine this claim for a Higgs mass in the range 125–126 GeV for which $\lambda_{\rm eff}(\mu)$ runs to very small values near the Planck scale and can significantly reduce the value of ξ required for inflation. Using the two-loop renormalization group equations and effective potential for Higgs ξ-inflation, we find that familiar inflationary solutions can have a non-minimal coupling as small as $\xi \sim 400$ without the potential developing a second minimum. We also find a new observationally allowed region of Higgs ξ-inflation with $\xi \sim 90$ and distinct inflationary predictions, including an observable level of the tensor-to-scalar ratio r.

KEYWORDS: Higgs Physics, Cosmology of Theories beyond the SM, Renormalization Group, Standard Model

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1 Introduction

A period of exponential expansion of the early universe driven by the potential energy of a scalar field — the inflaton — is an elegant explanation for the flatness, isotropy and homogeneity of the universe today [1–5]. Furthermore, it provides a very plausible mechanism for generating the nearly scale invariant spectrum of primordial density fluctuations that have been imprinted on the cosmic microwave background (CMB) [6, 7] and have grown into the large scale structure of galaxies [8]. The nature of the inflaton is, however, still unknown. While a large number of inflationary models that extend the scalar degrees of freedom of the Standard Model (SM) have been proposed (see e.g. [9, 10]), the possibility that the SM Higgs boson is the inflaton — a scenario attractive for its minimality — still remains for the model of Higgs inflation from a non-minimal coupling to gravity [11].[1]

This model of Higgs inflation, based on the work of [21–24], makes use of a large non-minimal gravitational coupling $\xi H^\dagger H \mathcal{R}$ between the Higgs doublet H and the Ricci scalar $\mathcal{R}$.[2] The effect of this coupling is to flatten the SM potential above the scale $M_{\rm Pl}/\sqrt{\xi}$, thereby allowing a sufficiently flat region for slow roll inflation. An analysis of the tree-level

[1]Other proposed models of Higgs inflation make use of special features of the SM potential that develop if the Higgs quartic coupling λ runs to very small values. The quasiflat SM potential considered in [12], however, predicts too large an amplitude of density fluctuations while false vacuum inflation [13–15] requires an additional scalar particle to achieve a graceful exit from inflation. Further possibilities, not discussed here, make use of derivative couplings of the Higgs to gravity or other non-renormalizable Higgs couplings [16–20].

[2]This is the only local, gauge-invariant interaction with mass dimension four or less that can be added to the SM once gravity is included.

potential finds $\xi \simeq 5 \times 10^4 \sqrt{\lambda}$ is required to produce the correct amplitude of primordial density fluctuations [11], which for $M_h \simeq 125$–$126\,\mathrm{GeV}$ [25] gives $\xi \sim 2 \times 10^4$. The predictions for the spectral index and the tensor-to-scalar ratio are also well within the current 1σ allowed regions [6, 7].

It has been pointed out, however, that Higgs ξ-inflation with the large value $\xi \sim 10^4$ suffers from a serious problem. Perturbative unitarity is violated at the scale M_{Pl}/ξ, and new physics entering at M_{Pl}/ξ to restore unitarity is naively expected to contain new particles and interactions that affect the potential in an uncontrollable way [26–29].[3] The self-consistency of the model in the inflationary region $h \gtrsim M_{\mathrm{Pl}}/\sqrt{\xi}$ is therefore questionable. To address the issue of unitarity violation while preserving the minimality of Higgs inflation, one must make a rather strong assumption that either additional non-renormalizable Higgs interactions accompany the non-minimal coupling and restore unitarity [34] or that new strong dynamics entering at M_{Pl}/ξ restores unitarity in a non-perturbative way [30, 31, 35, 36]. It is unknown whether the former approach can be made consistent with quantum corrections or the effect of additional potential and Yukawa interactions [33], while it is unclear whether strong coupling in graviton exchange processes for the latter scenario can unitarize scattering cross sections without requiring new physics [33]. If the latter scenario is possible, however, an approximate shift symmetry of the potential in the inflationary region $h \gtrsim M_{\mathrm{Pl}}/\sqrt{\xi}$ may keep quantum corrections to the potential under control [31].

The problem of perturbative unitarity violation in Higgs ξ-inflation, at least with regard to new physics entering at M_{Pl}/ξ below the inflationary scale, is perhaps not as severe as the tree-level estimate of ξ suggests. A Higgs mass $M_h \simeq 125$–$126\,\mathrm{GeV}$ is in the region that, for a top quark mass only about 2σ below its central value, the effective Higgs quartic coupling $\lambda_{\mathrm{eff}}(\mu)$ can run to very small (positive) values near the Planck scale [37–39]. The effect of small $\lambda_{\mathrm{eff}}(\mu)$ near the Planck scale is to reduce the value of ξ necessary for successful inflation [30, 35, 40] and hence push the scale of perturbative unitarity violation toward the inflationary scale. If inflation with $\xi \sim 1$ is possible for sufficiently small $\lambda_{\mathrm{eff}}(\mu)$ — a scenario that is not yet explored — the problem of perturbative unitarity violation occurring below the inflationary scale can be avoided.[4] Of course, an investigation of this possibility requires a proper treatment of the RG evolution and effective potential within the framework of Higgs ξ-inflation.

Extending the analysis of Higgs ξ-inflation to higher loop order is not entirely straightforward. While the renormalization group (RG) equations of the SM are perfectly adequate for describing the RG evolution below M_{Pl}/ξ, there are two ambiguities in the RG evolution

[3] It has been argued that the scale of perturbative unitarity violation for a large background Higgs field is higher than the small background field estimate M_{Pl}/ξ and, in particular, does not spoil the perturbative analysis of inflation [30–32]. In this case, one must make a non-trivial assumption about the new physics sector that the scale of new physics is background dependent [33]. In this paper, we make the more conservative working assumption that the scale of new physics is independent of the background Higgs field and therefore must be taken to be the lowest scale of perturbative unitarity violation, M_{Pl}/ξ.

[4] In this case, note that although the potential during inflation $V^{1/4} \lesssim 2 \times 10^{16}\,\mathrm{GeV}$ is constrained to be sub-Planckian [6, 7], the non-minimal coupling $\xi H^\dagger H \mathcal{R}$ with $\xi \sim 1$ is still relevant to inflation since the Higgs field $h \sim M_{\mathrm{Pl}}/\sqrt{\xi}$ is then assumed to be near the Planck scale.

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above $M_{\rm Pl}/\xi$ due to the non-minimal coupling of the Higgs. First, quantum loops involving the physical Higgs field (and not the Nambu-Goldstone bosons present in the Landau gauge) are heavily suppressed in this region [30, 35]. To deal with this, one can either use the chiral electroweak theory (SM with frozen radial Higgs mode) to derive the RG equations above $M_{\rm Pl}/\xi$ [30] or one can simply use the RG equations of the SM with a suppression factor for each Higgs running in a loop [35, 41–43]. Second, radiative corrections to the SM potential (in particular the choice of the renormalization scale $\mu(h)$) depend on whether they are computed in the Einstein or Jordan frame [40], and it is unclear which frame should be used without knowledge of physics at the Planck scale.

In this paper, we extend the two-loop analysis of Higgs ξ-inflation [30, 35] to include the three-loop SM beta functions for the gauge couplings [44] as well as the leading three-loop terms for the RG evolution of λ, the top Yukawa coupling y_t, and the Higgs anomalous dimension γ [45]. For the first time, a complete two-loop insertion of suppression factors for the *physical* Higgs loops, which was missing in [35], is carried out. The use of these RG equations provides a modest update to the previous analyses of Higgs ξ-inflation. The main focus of this paper, however, is to investigate the region of parameter space with $\lambda_{\rm eff}(\mu) \ll 1$ near the Planck scale that exists for the recently measured Higgs mass $M_h \simeq 125$–126 GeV and a top quark mass $M_t \sim 171$ GeV, about 2σ below its central value.[5]

The paper is organized as follows. In section 2, we give a brief review of Higgs ξ-inflation and the tree-level analysis. In section 3, the RG equations and the effective potential relevant for a two-loop analysis of Higgs ξ-inflation are presented. The numerical results and inflationary predictions for both the Einstein and Jordan frame renormalization prescriptions, with a particular focus on the small $\lambda_{\rm eff}^{\rm min}$ region, are given in section 4. A summary of the results and the conclusions are given in section 5.

2 Tree-level analysis

Let us first briefly review Higgs ξ-inflation and the tree-level computation of the inflationary predictions. Although the tree-level results will differ from those in the two-loop analysis, many qualitative features of the computation will remain the same.

As an example of inflation from a non-minimally coupled scalar, Higgs ξ-inflation is characterized by a non-minimal gravitational coupling $\xi H^\dagger H \mathcal{R}$ between the Higgs doublet H and the Ricci scalar $\mathcal{R}$. The Lagrangian of the model is given by [11]

$$\mathcal{L} = \mathcal{L}_{\rm SM} - \frac{M^2}{2}\mathcal{R} - \xi H^\dagger H \mathcal{R}, \tag{2.1}$$

where $\mathcal{L}_{\rm SM}$ is the SM Lagrangian and M is a mass parameter (the bare Planck mass) that can safely be identified with the present Planck mass value $M_{\rm Pl} = (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV for $\sqrt{\xi} \ll 10^{17}$. The part of (2.1) that is relevant to inflation gives the

[5]For the top quark mass central value, the SM potential develops an instability at around 10^{11} GeV [37, 38]. Since Higgs ξ-inflation requires the stability of the potential up to the inflationary scale $M_{\rm Pl}/\sqrt{\xi}$, one could interpret this result as disfavouring Higgs ξ-inflation at 2σ. The position advocated here is that a special region of Higgs ξ-inflation with $\lambda_{\rm eff}(\mu) \ll 1$ exists within only 2σ of experimental measurements.

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action

$$S_J = \int d^4x\sqrt{-g}\left[-\frac{M_{\rm Pl}^2}{2}\left(1+\frac{2\xi H^\dagger H}{M_{\rm Pl}^2}\right)\mathcal{R} + (\partial_\mu H)^\dagger (\partial^\mu H) - V\right], \tag{2.2}$$

where $V = \lambda\left(H^\dagger H - v^2/2\right)^2$ is the SM potential and the subscript J denotes the Jordan frame. This is the frame in which the inflationary model is defined.

To compute the inflationary observables, it is convenient to first remove the non-minimal coupling to gravity in (2.2) by performing the conformal transformation

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \qquad \Omega^2 = 1 + \frac{2\xi H^\dagger H}{M_{\rm Pl}^2}. \tag{2.3}$$

The resulting Einstein frame action is given by

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[-\frac{M_{\rm Pl}^2}{2}\tilde{\mathcal{R}} + \frac{1}{\Omega^2}(\partial_\mu H)^\dagger(\partial^\mu H) + \frac{3\xi^2}{\Omega^4 M_{\rm Pl}^2}\partial_\mu(H^\dagger H)\partial^\mu(H^\dagger H) - \frac{V}{\Omega^4}\right], \tag{2.4}$$

where $\tilde{\mathcal{R}}$ is calculated with the metric $\tilde{g}$. The action (2.4) simplifies greatly in the unitary gauge $H = \frac{1}{\sqrt{2}}\binom{0}{h}$, which may be used for the tree-level computation, giving

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[-\frac{M_{\rm Pl}^2}{2}\tilde{\mathcal{R}} + \frac{1}{2}\left(\frac{\Omega^2 + 6\xi^2h^2/M_{\rm Pl}^2}{\Omega^4}\right)\partial_\mu h\partial^\mu h - \frac{V}{\Omega^4}\right], \tag{2.5}$$

where $V = \frac{\lambda}{4}\left(h^2 - v^2\right)^2$ and $\Omega^2 = 1 + \xi h^2/M_{\rm Pl}^2$. It is also convenient to remove the non-canonical kinetic term for the Higgs field in (2.5) by changing to a new scalar field χ, defined by

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2h^2/M_{\rm Pl}^2}{\Omega^4}}. \tag{2.6}$$

The Einstein frame action then takes the form

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left[-\frac{M_{\rm Pl}^2}{2}\tilde{\mathcal{R}} + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - U(\chi)\right], \tag{2.7}$$

where the potential is given by

$$U(\chi) = \frac{V}{\Omega^4} = \frac{\lambda(h^2 - v^2)^2}{4(1+\xi h^2/M_{\rm Pl}^2)^2} \tag{2.8}$$

with $h = h(\chi)$. It is the flattening of the potential $U(\chi)$ to a constant value $U_0 \equiv \lambda M_{\rm Pl}^4/4\xi^2$ in the region $h \gtrsim M_{\rm Pl}/\sqrt{\xi}$ that allows slow roll inflation to occur.

The standard analysis of inflation in the slow roll approximation can be carried out for the field χ and potential $U(\chi)$. In the inflationary region $h^2 \gtrsim M_{\rm Pl}^2/\xi \gg v^2$, the slow roll parameters for $\xi \gg 1$ can be approximated by [35, 46] (see [23] for exact expressions)

$$\epsilon = \frac{M_{\rm Pl}^2}{2}\left(\frac{dU/d\chi}{U}\right)^2 \simeq \frac{4M_{\rm Pl}^4}{3\xi^2h^4}, \tag{2.9}$$

$$\eta = M_{\rm Pl}^2\frac{d^2U/d\chi^2}{U} \simeq \frac{4M_{\rm Pl}^4}{3\xi^2h^4}\left(1 - \frac{\xi h^2}{M_{\rm Pl}^2}\right), \tag{2.10}$$

$$\zeta^2 = M_{\rm Pl}^4\frac{\left(d^3U/d\chi^3\right)dU/d\chi}{U^2} \simeq \frac{16M_{\rm Pl}^6}{9\xi^3h^6}\left(\frac{\xi h^2}{M_{\rm Pl}^2} - 3\right). \tag{2.11}$$

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Slow roll ends when either $\epsilon \simeq 1$ or $|\eta| \simeq 1$. For (2.9) and (2.10), this occurs when $\epsilon \simeq 1$ at a field value $h_{\rm end} \simeq (4/3)^{1/4}\, M_{\rm Pl}/\sqrt{\xi} \simeq 1.07 M_{\rm Pl}/\sqrt{\xi}$. The number of e-folds of inflation as h changes from h_0 to $h_{\rm end}$ is given by [47]

$$N = \int_{h_{\rm end}}^{h_0} \frac{1}{M_{\rm Pl}^2} \frac{U}{dU/dh} \left(\frac{d\chi}{dh}\right)^2 dh \simeq \frac{3}{4}\left[\frac{h_0^2 - h_{\rm end}^2}{M_{\rm Pl}^2/\xi} + \ln\left(\frac{1+\xi h_{\rm end}^2/M_{\rm Pl}^2}{1+\xi h_0^2/M_{\rm Pl}^2}\right)\right]. \tag{2.12}$$

The values of the parameters (2.9)–(2.11) at a particular field value h_0, corresponding to the time at which the pivot scale $k_* \simeq 0.002 \mathrm{Mpc}^{-1}$ left the horizon during inflation, can be used to compare with the CMB data. This value of h_0 (or equivalently N) is a model-dependent quantity that is sensitive to the details of reheating. For Higgs ξ-inflation, an analysis of reheating finds that $N \simeq 59$, or equivalently $h_0 \simeq 9.14 M_{\rm Pl}/\sqrt{\xi}$, is the value at which k_* left the horizon during inflation [47, 48]. Using (2.9) in the WMAP9 normalization $U/\epsilon \simeq (0.0274 M_{\rm Pl})^4$ [6], the required value of ξ is[6]

$$\xi \simeq 48000\sqrt{\lambda} = 48000 \frac{M_h}{\sqrt{2}v} \simeq 17000. \tag{2.13}$$

The predictions for the spectral index n_s, the tensor-to-scalar ratio r, and the running of the spectral index $dn_s/d\ln k$ are given by

$$n_s = 1 - 6\epsilon + 2\eta \simeq 0.967, \tag{2.14}$$

$$r = 16\epsilon \simeq 0.0031, \tag{2.15}$$

$$\frac{dn_s}{d\ln k} = 24\epsilon^2 - 16\epsilon\eta + 2\zeta^2 \simeq 5.4 \times 10^{-4}. \tag{2.16}$$

These predictions for n_s and r are well within the current 1σ allowed regions from [6, 7], while the prediction of $dn_s/d\ln k$ is consistent with observations at the 1–2σ level.

3 Two-loop analysis

An analysis of Higgs ξ-inflation beyond the tree level must include both the running of the couplings and loop corrections to the (effective) potential [30, 35, 40]. The most significant effect of these higher order corrections comes from the running of the Higgs quartic coupling $\lambda = \lambda(\mu)$. For $M_h \simeq 125$–$126\,\mathrm{GeV}$, it is well known that the running of $\lambda(\mu)$ — or more specifically $\lambda_{\rm eff}(\mu)$ — causes the SM potential to develop an instability below the Planck scale unless the top quark mass is about 2σ below its central value [37, 38]. Since Higgs ξ-inflation requires the stability of the potential up to the inflationary scale $M_{\rm Pl}/\sqrt{\xi}$, in order to realize this model of inflation one must make the moderate assumption of a top quark mass $M_t \lesssim 171\,\mathrm{GeV}$. In this case, it has been shown that the small values of $\lambda_{\rm eff}(\mu)$ near the Planck scale can significantly reduce the non-minimal coupling ξ required for successful inflation [30, 35, 40].[7] The reason for this is relatively simple: the tree-level estimate (2.13)

[6]The *Planck* 2013 normalization $U/\epsilon \simeq (0.0269 M_{\rm Pl})^4$ [7] gives $\xi \simeq 18000$.

[7]Actually, the one-loop [40] and two-loop [30, 35] analyses predate the Higgs mass measurement and were carried out to determine the range of M_h allowed for Higgs ξ-inflation. In retrospect, however, a Higgs mass near the lower end of the allowed region suggests a value of $\xi \lesssim 10^3$ is required for successful inflation, with the lower limit of ξ unknown.

shows that it is the combination λ/ξ^2 that must be small ($\sim 4\times 10^{-10}$) in order to give the proper normalization of the CMB power spectrum. If $\lambda_{\rm eff}(\mu)$ is much smaller in the inflationary region than its tree-level value $\lambda \simeq 0.13$, then ξ must also be smaller than the tree-level estimate $\xi \simeq 18000$.

The smaller value of ξ required for successful inflation is particularly important since it is closely related to one of the most significant drawbacks of Higgs ξ-inflation: the violation of perturbative unitarity at the scale $M_{\rm Pl}/\xi$. For $\xi \to 1$, this scale is pushed toward the inflationary scale $M_{\rm Pl}/\sqrt{\xi}$ and the questionable assumptions of non-renormalizable operators [34] or new strong dynamics [30, 31, 35, 36] entering to restore unitarity are no longer required.[8] Since the lower limit of ξ in the case of small $\lambda_{\rm eff}(\mu)$ during inflation has not been explored, an important question is whether it is possible to realize Higgs ξ-inflation with $\xi \sim 1$ and hence avoid the perturbative unitarity issues with the model. Such a region is, by nature, highly sensitive to the running of $\lambda_{\rm eff}(\mu)$ and requires a proper loop analysis within the Higgs ξ-inflation framework.

To investigate the lower limit of ξ with $\lambda_{\rm eff}(\mu) \ll 1$ during inflation, we first describe the RG equations and the two-loop effective potential that are appropriate for Higgs ξ-inflation in sections 3.1 and 3.2, respectively. The analysis of inflation, including the lower limits on ξ and the inflationary predictions, are presented in section 4.

3.1 Renormalization group equations

The modification of the well-known RG equations of the SM for the Higgs ξ-inflation scenario has been discussed in [30, 35, 40–43]. Essentially, the scalar propagator of the physical Higgs field, which enters into loop diagram calculations for the RG equations, must be multiplied by the field-dependent factor [35, 42][9]

$$s(h) = \frac{1+\xi h^2/M_{\rm Pl}^2}{1+(1+6\xi)\xi h^2/M_{\rm Pl}^2}. \tag{3.1}$$

For small field values $h \ll M_{\rm Pl}/\xi$, $s \simeq 1$ and the RG equations for the SM are perfectly adequate for describing the RG evolution. For large field values $h \gg M_{\rm Pl}/\xi$, however, the physical Higgs propagator is suppressed by a factor $s \simeq 1/(1+6\xi)$ and hence the RG equations differ from those of the SM. Two methods of dealing with this effect have been considered in the literature [30, 35], leading to somewhat different results.

The first method of treating the suppressed Higgs loops, which is described in [35], is to insert one suppression factor s into the RG equations of the SM for each off-shell Higgs propagator. Originally this was done by extracting out all Higgs doublet propagators at one-loop order and inserting the appropriate factors of s, repeating the process only

[8] Of course, the Higgs field h during inflation becomes trans-Planckian in this case and one must worry about the effects of higher dimensional operators suppressed by the Planck scale, which may spoil the flatness of the potential or the inflationary predictions [47]. As remarked in [35], however, the same worry applies to many minimal models of inflation, such as $m^2\phi^2$ chaotic inflation.

[9] The reason for this suppression is that the canonical momentum of h (which is evaluated in the Einstein frame with a canonical gravity sector) gives a non-standard commutator $[h(\vec{x}), \dot{h}(\vec{y})] = i\hbar s(h)\delta^3(\vec{x}-\vec{y})$ in the Jordan frame after imposing the standard commutation relations $[h(\vec{x}), \pi(\vec{y})] = i\hbar\delta^3(\vec{x}-\vec{y})$.

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for obvious terms at two-loop order [35]. It was later pointed out, however, that only the propagator of the physical Higgs field and not the Nambu-Goldstone bosons that are present in the Landau gauge should come with such a factor [30]. The corrected RG equations with systematic insertions of s for all two-loop terms, except for β_λ, are given in [43]. By using these RG equations in the full two-loop SM effective potential from [38] (with $m^2 \to 0$ and $M_h^2 \to 3s\lambda h^2$) and demanding that the potential be independent of μ, we have been able to extract the two-loop part for β_λ.[10] A similar procedure can then be used to obtain the two-loop RG equation for the Higgs mass parameter m^2 (in the notation of [38]) with appropriate suppression factors. Although β_{m^2} is not actually required for an analysis of Higgs ξ-inflation, it can be used to derive the RG equation for ξ through the relation $\beta_\xi = (\xi + 1/6)\,\gamma_m$, where $\gamma_m = \beta_{m^2}/m^2$ [42]. The complete set of two-loop RG equations with suppressed physical Higgs loops is given in appendix A.

The second method of treating the suppressed Higgs loops is to instead view the effect as a suppression of the effective Higgs coupling to other SM fields and, for large ξ, neglect the physical Higgs field altogether in the region $h \gtrsim M_{\rm Pl}/\xi$ [30]. The resulting theory (SM with frozen radial Higgs mode) is known as the chiral electroweak theory and has been studied previously in the literature. It is therefore possible to extract one-loop RG equations, which are valid for $\xi \gg 1$, from earlier works such as [49]. In [30], however, the RG equations for λ, y_t, and ξ derived in this way differ from (A.1), (A.2), and (A.6) with $s = 0$. A closer look reveals two sources (though not necessarily errors) for this discrepancy. First, the equation for the running of v^2 used in [30], which was first given in [49], differs from the running of the SM Higgs field h^2 in the Landau gauge,

$$16\pi^2 \mu \frac{\partial}{\partial\mu} h^2 = \left(\frac{3}{2} g'^2 + \frac{9}{2} g^2 - 6 y_t^2\right) h^2. \tag{3.2}$$

In the usual SM case, the running of the Higgs vacuum expectation value v^2 and the Higgs field h^2 are both gauge-dependent quantities [50] and have the same running [51]. Although it has been argued that v^2 in the chiral electroweak theory is a gauge-invariant parameter and therefore its running should also be gauge invariant, we have found it difficult to reproduce the chiral electroweak theory result using Feynman diagrams and understand why its running differs from the running of h^2 in the SM. In any case, if eq. (5.6) of [30] is replaced by the similar expression (3.2), the resulting one-loop equation for β_{y_t} agrees with (A.2) for $s = 0$. Second, β_λ is derived in [30] by demanding that the one-loop effective potential be independent of μ, where the one-loop potential does not include the usual contribution from the Nambu-Goldstone bosons [38]

$$\Delta U_1 = \frac{3 M_G^4}{64\pi^2}\left(\ln \frac{M_G^2}{\mu^2} - \frac{3}{2}\right), \tag{3.3}$$

where $M_G^2 = \lambda h^2$. Although the Goldstone boson contribution to the effective potential is strongly suppressed for prescription I (see section 3.2), the result of excluding this term in

[10] In the process, we believe that two typos in the complete expression for the two-loop SM effective potential have been discovered. In [38], the final term of (A.3) should read $-\chi I_{ttg}$ instead of $-\chi I_{ttz}$ and the second last term on the third line of (A.5) should read $-3I_{w00}$ instead of $+I_{w00}$.

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deriving β_λ is equivalent to suppressing some Feynman diagrams with off-shell Goldstone boson propagators; that is, the $\left(6 + 18s^2\right)\lambda^2$ term in (A.1) disappears entirely. While this difference is small in the region $h \gtrsim M_{\rm Pl}/\xi$ (numerically it is smaller than the two-loop correction to β_λ), if the contribution (3.3) is included in eq. (4.1) of [30] the resulting one-loop equation for β_λ agrees with (A.1) for $s = 0$. Note that the one-loop equation for β_ξ in [30] still differs from (A.6) even after accounting for these changes. Specifically, the latter has a factor of $\xi + 1/6$ instead of ξ and an additional term $(6 + 6s)\,\lambda$ compared to the former. Again, these differences in β_ξ are small (typically below the size of the two-loop correction to β_ξ) since we always have $\xi \gg 1/6$ and since λ is small in the region $M_{\rm Pl}/\xi \lesssim h \lesssim M_{\rm Pl}/\sqrt{\xi}$ over which ξ runs.

It is also worth mentioning that the second method includes the effects of additional counterterms taken from the chiral electroweak theory [49] that arise to cancel divergences in the non-renormalizable SM sector without a Higgs field. These effects appear through the additional couplings α_0 and α_1 that modify the renormalization of the Z boson mass [30] and contribute to the effective potential at the two-loop level. Numerically, however, the Z boson mass contribution at the two-loop level, and hence this effect, is subleading [38].

The two methods of treating the suppressed Higgs loops for $h \gtrsim M_{\rm Pl}/\xi$ are therefore quite similar, at least for large ξ. The first method uses s factors to smoothly interpolate between the SM-like RG evolution at low energies and the RG evolution with suppressed physical Higgs propagators at high energies, while the second method models this transition as an abrupt change at $M_{\rm Pl}/\xi$.[11] Since the s-factor treatment also handles the case $\xi \sim 1$, though, we adopt the first method [35, 41–43] and use a suppression factor s for each off-shell Higgs propagator in the SM RG equations for our analysis of Higgs ξ-inflation. Despite the different treatments of the RG equations described above, the numerical differences are small enough that a two-loop analysis in the region $h \gtrsim M_{\rm Pl}/\xi$ should be justified.

With the recent SM calculation of the three-loop beta functions for the gauge couplings [44] and the leading three-loop terms for β_λ, β_{y_t}, and γ [45], it is relatively simple to include these contributions in the RG equations (A.1)–(A.7) so that the Higgs ξ-inflation analysis matches the NNLO analysis of [38] for $h \lesssim M_{\rm Pl}/\xi$ (see appendix A). Note that we do not attempt to insert the appropriate factors of s into these expressions since the corrections would be smaller than the uncertainty in the RG equations for $h \gtrsim M_{\rm Pl}/\xi$.

For a complete description of the RG evolution in Higgs ξ-inflation, the equations (A.1)–(A.7) with the three-loop corrections (A.8)–(A.13) must be supplemented by values of the SM couplings at the electroweak scale and the value of ξ at some high scale, say $M_{\rm Pl}/\xi$. Appropriate initial values for the SM couplings can be found in [38]. For the central values of $\alpha_Y^{-1}(M_Z)$ and $\alpha_2^{-1}(M_Z)$, the initial values of the gauge couplings g' and g are

$$g'(M_Z) = \sqrt{\frac{4\pi}{98.35}} \simeq 0.3575, \tag{3.4}$$

$$g(M_Z) = \sqrt{\frac{4\pi}{29.587}} \simeq 0.65171, \tag{3.5}$$

[11] A smooth interpolation is preferred, but for $\xi \gg 1$ the function $s(h)$ changes rapidly in the region $h \sim M_{\rm Pl}/\xi$ and so the modification of the numerics is negligible [30].

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where $M_Z = 91.1876\,\mathrm{GeV}$. For the strong gauge coupling g_s, the initial value depends more sensitively on the uncertainty in $\alpha_s(M_Z) = 0.1184 \pm 0.0007$. It is given by

$$g_s(M_t) = 1.1645 + 0.0031 \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) - 0.00046 \left(\frac{M_t}{\mathrm{GeV}} - 173.15 \right), \tag{3.6}$$

where M_t is the top quark pole mass determined from experiment. The initial value of the top quark Yukawa coupling y_t is

$$\begin{aligned} y_t(M_t) = 0.93587 + 0.00557 \left(\frac{M_t}{\mathrm{GeV}} - 173.15 \right) - 0.00003 \left(\frac{M_h}{\mathrm{GeV}} - 125 \right) \\ - 0.00041 \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 0.00200_{\mathrm{th}}, \end{aligned} \tag{3.7}$$

where M_h is the Higgs pole mass, while the initial value of the Higgs quartic coupling λ is

$$\lambda(M_t) = 0.12577 + 0.00205 \left(\frac{M_h}{\mathrm{GeV}} - 125 \right) - 0.00004 \left(\frac{M_t}{\mathrm{GeV}} - 173.15 \right) \pm 0.00140_{\mathrm{th}}. \tag{3.8}$$

Note that the theoretical uncertainty for $\lambda(M_t)$ in (3.8) is equivalent to an uncertainty in the Higgs pole mass of $\pm 0.7\,\mathrm{GeV}$ [38]. This means, in particular, that for the measured Higgs mass $M_h = 125.7 \pm 0.4$ [52] it is quite reasonable to use values of $M_h \simeq 124$–$127\,\mathrm{GeV}$ in (3.7) and (3.8). For the non-minimal coupling ξ, we are (a priori) free to choose its initial value ξ_0 at some high scale. We take the scale to be M_{Pl}/ξ_0 so that, by definition,

$$\xi(M_{\mathrm{Pl}}/\xi_0) = \xi_0. \tag{3.9}$$

The RG equations (A.1)–(A.13) with the initial values (3.4)–(3.9) are therefore the ones we use to describe the RG evolution of the couplings for Higgs ξ-inflation.

3.2 Two-loop effective potential

The effective potential for Higgs ξ-inflation, like the RG equations, differs from the well-known SM result [38, 53] due to the suppression of the physical Higgs propagators. As described in [40], however, the effective potential cannot be fixed unambiguously; there are two inequivalent renormalization prescriptions depending on whether quantum corrections to the potential are computed in the Einstein frame (prescription I) [11] or the Jordan frame (prescription II) [54]. Without knowing the behaviour of the quantum theory at the Planck scale, it is unclear which prescription should be used. The former prescription has been connected to ideas of a possible quantum scale invariance [55–57] while [54] has argued that the latter prescription is correct because the Jordan frame is the one in which physical distances are measured.

For sufficiently large $\lambda_{\mathrm{eff}}(\mu)$, the running of $\lambda_{\mathrm{eff}}(\mu)$ during inflation is small and the choice of renormalization prescription is irrelevant from a practical point of view [30, 38, 40]. For the small $\lambda_{\mathrm{eff}}(\mu)$ allowed by the recent Higgs mass measurement, however, the choice of renormalization prescription can significantly affect the behaviour of $\lambda_{\mathrm{eff}}(\mu)$ and hence the potential during inflation. Both renormalization prescriptions must therefore be considered.

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For prescription I, the tree-level SM potential

$$V_0(h) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2 \simeq \frac{\lambda}{4}h^2 \tag{3.10}$$

is first rewritten in the Einstein frame ($h = h(\chi)$) using (2.8), giving

$$U_0(\chi) = \frac{\lambda h^4}{4\Omega^4}. \tag{3.11}$$

Note that the v^2 term in (3.10) has been safely neglected in the inflationary region $h^2 \gtrsim M_{\rm Pl}^2/\xi \gg v^2$. The one-loop radiative corrections induced by the fields of the SM then take the Coleman-Weinberg form [38][12]

$$\begin{aligned} U_1(\chi) = \frac{1}{16\pi^2}\Bigg[&\frac{3M_W^4}{2}\left(\ln\frac{M_W^2}{\mu^2} - \frac{5}{6}\right) + \frac{3M_Z^4}{4}\left(\ln\frac{M_Z^2}{\mu^2} - \frac{5}{6}\right) - 3M_t^4\left(\ln\frac{M_t^2}{\mu^2} - \frac{3}{2}\right) \\ &+ \frac{M_h^4}{4}\left(\ln\frac{M_h^2}{\mu^2} - \frac{3}{2}\right) + \frac{3M_G^4}{4}\left(\ln\frac{M_G^2}{\mu^2} - \frac{3}{2}\right)\Bigg], \end{aligned} \tag{3.12}$$

where the particle masses M_W, M_Z, M_t, M_h, and M_G are computed from the tree-level potential (3.11), giving [11, 40, 60][13]

$$\begin{aligned} &M_W^2 = \frac{g^2 h^2}{4\Omega^2}, && M_Z^2 = \frac{\left(g^2 + g'^2\right) h^2}{4\Omega^2}, \qquad M_t^2 = \frac{y_t^2 h^2}{2\Omega^2}, \\ &M_h^2 = \frac{3s\lambda h^2}{\Omega^4}\left(\frac{1 - \xi h^2/M_{\rm Pl}^2}{1 + \xi h^2/M_{\rm Pl}^2}\right), && M_G^2 = \frac{\lambda h^2}{\Omega^4}. \end{aligned} \tag{3.13}$$

Note that the particle masses M_W^2, M_Z^2, and M_t^2 in (3.13) differ from the flat space results by the conformal factor $\Omega^2 = 1 + \xi h^2/M_{\rm Pl}^2$ that appears in the denominator, while the physical Higgs mass M_h^2 and Goldstone boson mass M_G^2 contain additional factors. With the exception of the suppression factor $s = s(h)$ in the physical Higgs mass, the appearance of these additional factors in M_h^2 and M_G^2 is due to using the asymptotically flat tree-level potential (3.11) to determine particle masses rather than the Jordan frame potential (3.10). These additional factors lead to a suppression of the physical Higgs and Goldstone boson contributions to the effective potential (relative to those from W, Z, and t) during inflation for prescription I, as found in [11, 40, 60].

The two-loop radiative corrections $U_2(\chi)$ can easily be found by using the modified particle masses (3.13) in the two-loop SM result of [38], but due to the rather long and unenlightening form of this expression we do not reproduce it here. The RG-improved effective potential is then determined from $U_{\rm eff}(\chi) = U_0 + U_1 + U_2$ in the usual way by

[12] Up to corrections from the time-dependence of the background Higgs field as it rolls down its potential. Such corrections have been considered in [58, 59] for the simpler Abelian Higgs model but have not yet been studied for the Higgs ξ-inflation model. Analyzing these corrections goes beyond the scope of this paper.

[13] We obtain this result by expanding $H = \frac{1}{\sqrt{2}}\binom{0}{h} + \binom{\hat{G}^+}{(\hat{h}+i\hat{G}^0)/\sqrt{2}}$ in the full expression for the tree-level potential $U_0 = \lambda(H^\dagger H)^2/\Omega^4 = \lambda(H^\dagger H)^2/(1 + 2\xi H^\dagger H/M_{\rm Pl}^2)^2$ to quadratic order in the fields $\hat{h}$, $\hat{G}^+$, $\hat{G}^0$, where h is the classical background value of the Higgs field $\hat{h}$ [60].

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using the RG equations from appendix A to run the couplings and making the replacement $h \to e^{\Gamma(\mu)} h$, where

$$\Gamma(\mu) = -\int_{M_t}^{\mu} \gamma(\mu') d \ln \mu' \tag{3.14}$$

and $\gamma = -d \ln h / d \ln \mu$ is the anomalous dimension of the Higgs field [61].[14] The effective Higgs quartic coupling $\lambda_{\text{eff}}(\mu)$ is then defined through

$$U_{\text{eff}}(\chi) \equiv \frac{\lambda_{\text{eff}}(\mu) h^4}{4 \Omega^4}, \tag{3.15}$$

where all couplings in (3.15) are evaluated at some renormalization scale μ. The dependence of the effective potential on the scale μ is spurious, but to minimize the logarithms from higher loop corrections it is appropriate to take $\mu = \kappa h / \Omega$ proportional to the background mass of a vector boson or top quark [30]. For simplicity, we choose the constant of proportionality to be $\kappa = 1$.

For prescription II, quantum corrections to the potential (3.10) are computed in the Jordan frame before transforming to the Einstein frame. In this case, the one-loop radiative corrections to the effective potential take the form [42]

$$\begin{aligned} U_1(\chi) = \frac{1}{16\pi^2 \Omega^4} \bigg[& \frac{3M_W^4}{2} \left(\ln \frac{M_W^2}{\mu^2} - \frac{5}{6} \right) + \frac{3M_Z^4}{4} \left(\ln \frac{M_Z^2}{\mu^2} - \frac{5}{6} \right) - 3M_t^4 \left(\ln \frac{M_t^2}{\mu^2} - \frac{3}{2} \right) \\ & + \frac{M_h^4}{4} \left(\ln \frac{M_h^2}{\mu^2} - \frac{3}{2} \right) + \frac{3M_G^4}{4} \left(\ln \frac{M_G^2}{\mu^2} - \frac{3}{2} \right) \bigg], \end{aligned} \tag{3.16}$$

where the particle masses M_W^2, M_Z^2, M_t^2, M_h^2, and M_G^2 appear without the conformal factor Ω^2 or additional factors in their denominators,

$$\begin{aligned} M_W^2 &= \frac{g^2 h^2}{4}, \qquad M_Z^2 = \frac{\left(g^2 + g'^2\right) h^2}{4}, \qquad M_t^2 = \frac{y_t^2 h^2}{2}, \\ M_h^2 &= 3 s \lambda h^2, \qquad M_G^2 = \lambda h^2. \end{aligned} \tag{3.17}$$

The two-loop radiative corrections $U_2(\chi)$ can be found by using the particle masses (3.17) in the two-loop SM result [38] and dividing the expression by the conformal factor Ω^4. The effective Higgs quartic coupling $\lambda_{\text{eff}}(\mu)$ is again defined through (3.15), but in this case taking the renormalization scale to be proportional to the background mass of a vector boson or top quark requires $\mu = \kappa h$. For simplicity, we again choose $\kappa = 1$.

In practice, the most significant difference between the effective potentials for the two renormalization prescriptions is the functional dependence $\mu = \mu(h)$.[15] For prescription I, $\mu = h/\Omega$ approaches a constant value in the inflationary region $h \gtrsim M_{\text{Pl}}/\sqrt{\xi}$ (and hence so do the couplings $g(\mu)$, $g'(\mu)$, etc. in (3.13)) while for prescription II the renormalization scale $\mu = h$ does not. As a result, the effective potential for prescription I approaches a

[14]Note the difference in sign between the definition of γ here and the definition of γ in [38].

[15]The additional suppression of the physical Higgs and Goldstone boson masses for prescription I is relatively minor since these masses, and hence their contributions to the effective potential, are small compared to M_W, M_Z, and M_t for the small λ in the inflationary region.

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constant value in the inflationary region (even after including radiative corrections) while the effective potential for prescription II, due to the continued running of the couplings, does not. This difference, as we will see, can have a large impact on Higgs ξ-inflation and its predictions for small $\lambda_{\rm eff}(\mu)$.

4 Numerical results

For a fixed Higgs mass M_h, top quark mass M_t, strong coupling $\alpha_s(M_Z)$, and non-minimal coupling ξ_0, it is straightforward to numerically solve the RG equations (A.1)–(A.13) with initial conditions (3.4)–(3.9) and use the effective potential $U(\chi)$ (for either prescription I or II) to compute the inflationary parameters. However, since the focus of this paper is on the region of parameter space with $\lambda_{\rm eff}(\mu) \ll 1$, we instead replace the parameter M_t in favour of $\lambda_{\rm eff}^{\rm min} \equiv \min\{\lambda_{\rm eff}(\mu)\}$. Intuitively, this can be understood as adjusting the top quark mass M_t to yield the desired $\lambda_{\rm eff}^{\rm min}$ for a fixed choice of M_h, $\alpha_s(M_Z)$, and ξ_0. Figure 1 shows that the special region $\lambda_{\rm eff}^{\rm min} \simeq 0$ exists for a top quark mass $M_t \sim 171\,\text{GeV}$ about 2–3σ below its central value. Since values of $\lambda_{\rm eff}^{\rm min} \sim 0.01$ are typical within the experimental and theoretical uncertainty of the various parameters, a fine-tuning of some combination of parameters is necessary to achieve $0 < \lambda_{\rm eff}^{\rm min} \lesssim 0.01$.[16] Note that negative values of $\lambda_{\rm eff}^{\rm min}$, as well as sufficiently small positive values, cause the effective potential to develop a second minimum below the inflationary scale and hence spoil Higgs ξ-inflation. We therefore restrict ourselves to the region $0 < \lambda_{\rm eff}^{\rm min} \lesssim 0.01$ in which the effective potential is stable.

The non-minimal coupling ξ_0 is not actually a free parameter, of course, but must be chosen to give the correct normalization of the CMB power spectrum (see section 2). For a fixed M_h and $\alpha_s(M_Z)$, the procedure for determining the inflationary predictions for a particular choice of $\lambda_{\rm eff}^{\rm min}$ and renormalization prescription is as follows:

1. Choose a value of ξ_0. Adjust the top quark mass M_t to give the desired value of $\lambda_{\rm eff}^{\rm min}$ when solving the RG equations. For $\lambda_{\rm eff}^{\rm min} \lesssim 0.01$, this may involve fine-tuning M_t.

2. Use the effective potential $U(\chi)$ (for prescription I or II) to compute the inflationary parameters and determine $U(h_0)/\epsilon(h_0)$ at a field value h_0 corresponding to $N = 59$ e-folds before the end of inflation.

3. Repeat the steps above for different values of ξ_0 until the correct normalization $U/\epsilon \simeq (0.0274 M_{\rm Pl})^4$ is achieved.[17]

4. Compute the inflationary predictions for the spectral index n_s, the tensor-to-scalar ratio r, and the running of the spectral index $dn_s/d\ln k$.

We discuss the numerical results for prescriptions I and II separately.

[16] In [39] it is argued that a UV fixed point in an asymptotically safe theory of gravity may ensure very small values of $\lambda(\mu)$ near the Planck scale. In this case, fine-tuning may only be necessary for values of $\lambda_{\rm eff}^{\rm min}$ smaller than the typical size of the shift in $\lambda_{\rm eff}(\mu)$ due to radiative corrections to the effective potential, $\delta\lambda_{\rm eff}(\mu \sim M_{\rm Pl}) \sim 4 \times 10^{-4}$.

[17] Note that prescription II with sufficiently small $\lambda_{\rm eff}^{\rm min}$ can have two solutions for ξ_0. The large ξ_0 solution, however, predicts $n_s > 1.02$ and is therefore inconsistent with observations [6, 7].

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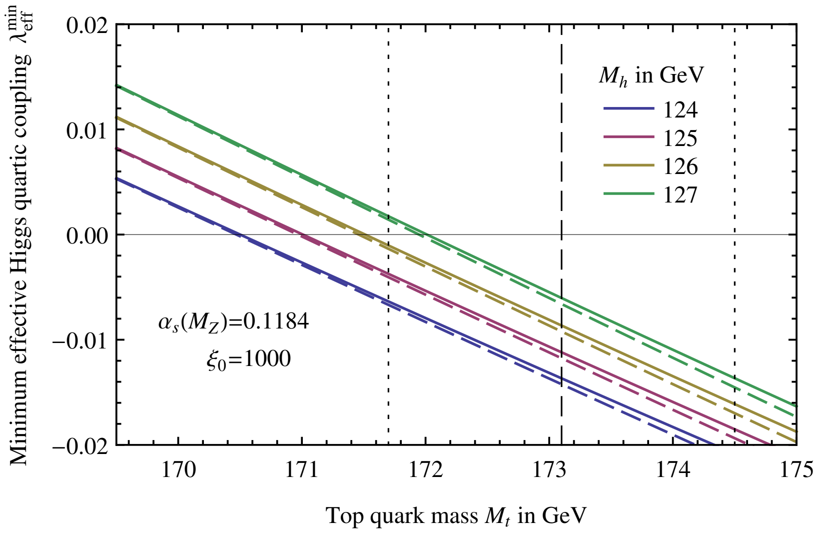

Figure 1. Values of $\lambda_{\rm eff}^{\rm min}$ as a function of M_t for fixed $\alpha_s(M_Z) = 0.1184$ and $\xi_0 = 1000$. The four solid (dashed) curves correspond to a Higgs mass M_h of 124, 125, 126, and 127 GeV from bottom to top for renormalization prescription I (II). The vertical dashed and dotted lines give the central value and $\pm 2\sigma$ range for M_t [38]. A shift in $\alpha_s(M_Z)$ of $\pm 1\sigma$ (± 0.0007) roughly corresponds to a shift in M_h of ± 0.5 GeV while changing ξ_0 by an order of magnitude has little effect.

4.1 Inflationary predictions for prescription I

For prescription I, the results for ξ_0 and the inflationary predictions for n_s and r (as a function of $\lambda_{\rm eff}^{\rm min}$) are presented in figure 2. The running of the spectral index $dn_s/d\ln k$ always remains small, within the range $(5.0\text{–}5.6) \times 10^{-4}$.

Let us first discuss the non-minimal coupling ξ_0. Figure 2 shows that the value of ξ_0 required for the CMB normalization deviates from the tree-level estimate $\xi_0 \simeq 48000\sqrt{\lambda_{\rm eff}^{\rm min}}$ as $\lambda_{\rm eff}^{\rm min}$ decreases below about 10^{-4}. In particular, ξ_0 reaches a minimum value of $\xi_0 \sim 400$ at $\lambda_{\rm eff}^{\rm min} \sim 10^{-4.4}$ and then begins to increase. This behaviour can be traced to the rapid decrease in ϵ (and hence the tensor-to-scalar ratio $r = 16\epsilon$) over this range, which causes U/ϵ to increase despite smaller values of $\lambda_{\rm eff}^{\rm min}$. A larger non-minimal coupling ξ_0 is therefore required to give the correct CMB normalization. This result demonstrates that the sharp decrease in ξ_0 seen in [40] and figure 4 of [30] for prescription I does not continue indefinitely but only allows ξ_0 as small as about 400. The violation of perturbative unitarity at the scale $M_{\rm Pl}/\xi_0 \ll M_{\rm Pl}/\sqrt{\xi_0}$ therefore remains a problem for Higgs ξ-inflation in the small $\lambda_{\rm eff}^{\rm min}$ region. For sufficiently small $\lambda_{\rm eff}^{\rm min}$ (e.g. $\lesssim 10^{-4.6}$), no solutions for ξ_0 are possible since the effective potential develops a second minimum and hence spoils the Higgs ξ-inflation scenario.[18]

Figure 2 also shows small deviations in the inflationary predictions for the spectral index n_s and the tensor-to-scalar ratio r. As $\lambda_{\rm eff}^{\rm min}$ decreases below about $10^{-3.5}$, the

[18] In this case, the effective potential rises to a local maximum and then decreases slowly to a constant value as $h \to \infty$. The shape of the potential may be suitable for a sort of false vacuum inflation in which the Higgs field can start with any value $h \gtrsim M_{\rm Pl}/\sqrt{\xi}$, but an analysis of this case goes beyond the scope of this paper.

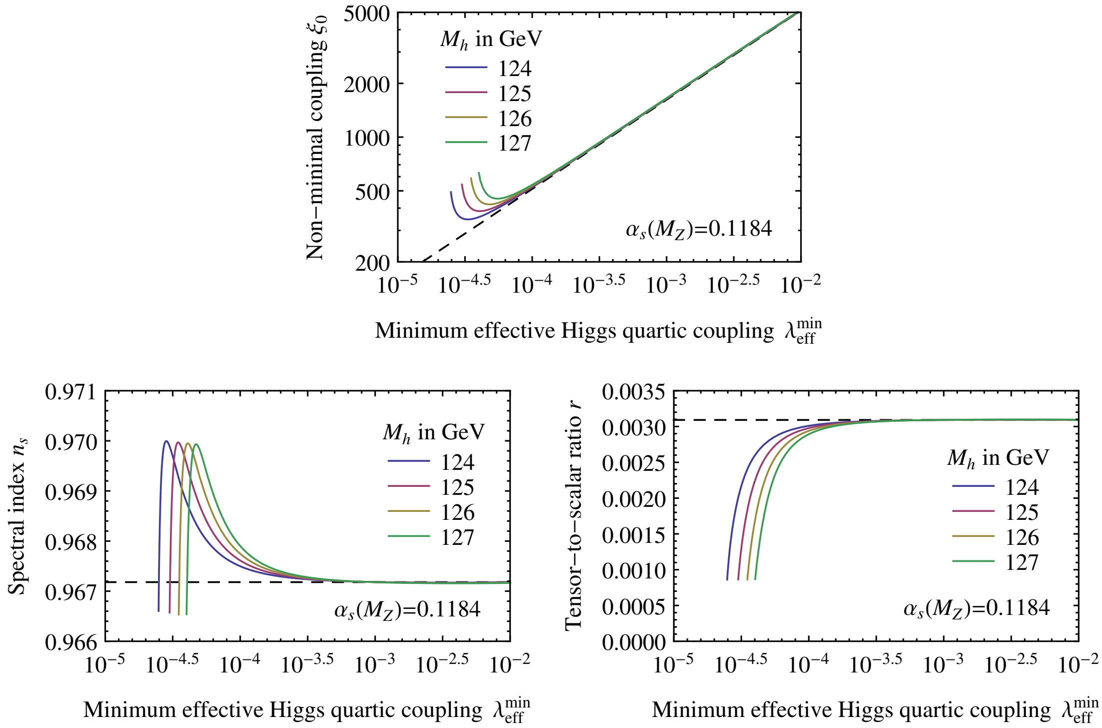

Figure 2. Numerical results for the non-minimal coupling ξ_0 and inflationary predictions for the spectral index n_s and the tensor-to-scalar ratio r as a function of $\lambda_{\rm eff}^{\rm min}$ for prescription I. The four solid curves correspond to a Higgs mass M_h of 124, 125, 126, and 127 GeV from left to right while the dashed lines give the tree-level predictions. A shift in $\alpha_s(M_Z)$ of $\pm 2\sigma$ (± 0.0014) roughly corresponds to a shift in M_h of ∓ 0.5 GeV. Changing the number of e-folds from $N = 59$ to 62 shifts the tree-level predictions by a small (calculable) amount but does not change the qualitative behaviour of the curves about the tree-level predictions.

spectral index rises to about 0.970 from its tree-level prediction of 0.967 before decreasing rapidly, while the tensor-to-scalar ratio drops quickly below its tree-level prediction of 0.0031. Although a similarly rapid change in n_s and r can be seen in [30, 40] as the Higgs mass approaches values corresponding to $\lambda_{\rm eff}^{\rm min} \simeq 0$, the results presented here (as a function of $\lambda_{\rm eff}^{\rm min}$) provide a much clearer picture of Higgs ξ-inflation in this now experimentally favoured region. From a practical point of view, we see that the deviations of n_s and r from the tree-level predictions are sufficiently small that they would be difficult to distinguish from the tree-level results observationally. Consequently, for all allowed values $10^{-4.6} \lesssim \lambda_{\rm eff}^{\rm min} \lesssim 10^{-2}$, the predictions for n_s and r are well within the current 1σ limits [6, 7]. The small prediction for $dn_s/d\ln k \sim 5 \times 10^{-4}$ is also consistent with observations at the 1–2σ level [6, 7].

4.2 Inflationary predictions for prescription II

For prescription II, there are two disjoint regions of $\lambda_{\rm eff}^{\rm min}$ that can lead to acceptable inflation: one with larger values $\lambda_{\rm eff}^{\rm min} \gtrsim 10^{-3.3}$–$10^{-2.3}$ (depending on M_h) and one with smaller values $\lambda_{\rm eff}^{\rm min} \sim 10^{-4}$. The results for ξ_0 and the inflationary predictions for n_s and r are quite different for these two regions and are presented in figures 3 and 4, respectively.

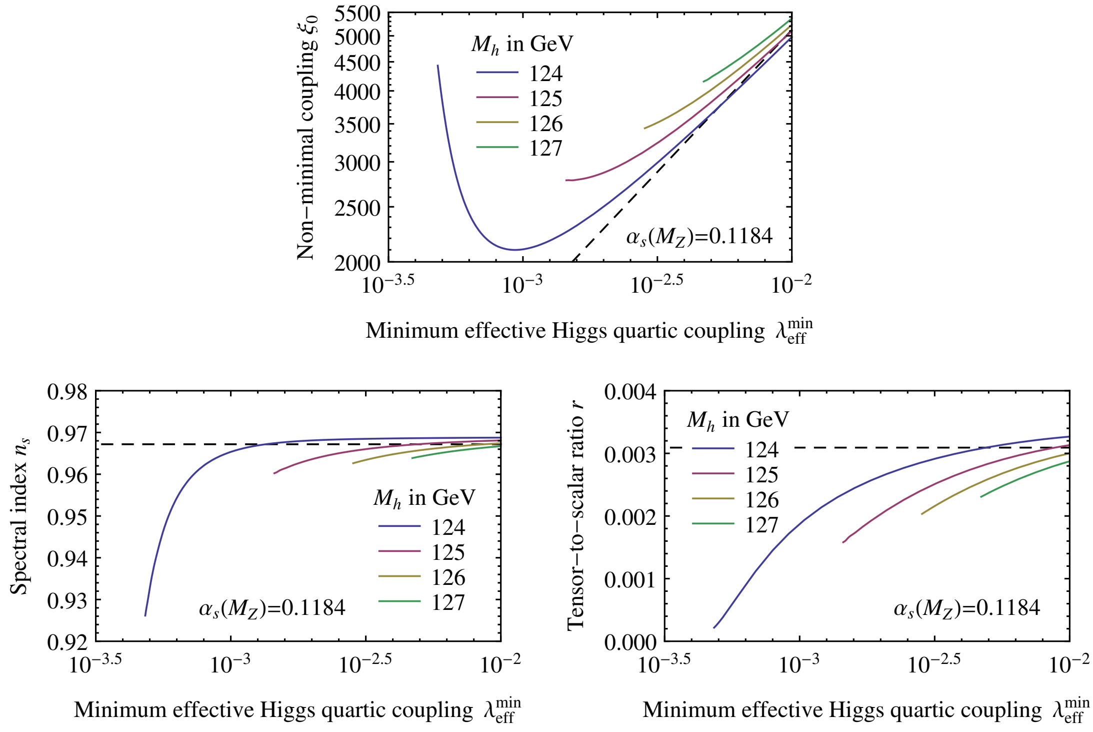

Figure 3. Numerical results for the non-minimal coupling ξ_0 and inflationary predictions for the spectral index n_s and the tensor-to-scalar ratio r in the larger $\lambda_{\rm eff}^{\rm min}$ region for prescription II. The four solid curves correspond to a Higgs mass M_h of 124, 125, 126, and 127 GeV from left to right while the dashed lines give the tree-level predictions. A shift in $\alpha_s(M_Z)$ of $\pm 2\sigma$ (± 0.0014) roughly corresponds to a shift in M_h of ∓ 0.5 GeV. Changing the number of e-folds from $N = 59$ to 62 shifts the tree-level predictions by a small (calculable) amount but does not change the qualitative behaviour of the curves about the tree-level predictions.

Let us first consider the region of larger values of $\lambda_{\rm eff}^{\rm min}$, which is the only one that has been considered previously in the literature [30, 35, 40]. Figure 3 shows that the required value of ξ_0 in this region behaves similarly to that of prescription I except that the minimum value of ξ_0 — if it can be reached without the potential developing a second minimum — occurs at larger $\lambda_{\rm eff}^{\rm min}$ (i.e. $\lambda_{\rm eff}^{\rm min} \gtrsim 10^{-3}$). This difference is due to the stronger effect of the running of $\lambda_{\rm eff}(\mu)$ for prescription II. Specifically, the running of $\lambda_{\rm eff}(\mu)$ to its minimum value overcomes the flattening of the potential in the inflationary region more quickly than for prescription I, and hence causes the effective potential to develop a second minimum for more moderate values of $\lambda_{\rm eff}^{\rm min}$. As a result, a non-minimal coupling only as small as $\xi_0 \sim 2000$–4000 (depending on M_h) is allowed for this region of $\lambda_{\rm eff}^{\rm min}$. Similar lower limits for ξ_0, as well as the qualitative rise in ξ_0 as $\lambda_{\rm eff}^{\rm min}$ decreases, have been found in [30, 40] for prescription II. Again, these values of ξ_0 are not small enough to prevent the perturbative unitarity violation at $M_{\rm Pl}/\xi_0$ from occurring well below the inflationary scale.

Figure 3 also shows that the predictions for the spectral index n_s and the tensor-to-scalar ratio r *decrease* from their tree-level values as $\lambda_{\rm eff}^{\rm min} \to 0$. The decrease observed here (similar to prescription I) is consistent with the results of [30, 40] rather than with the

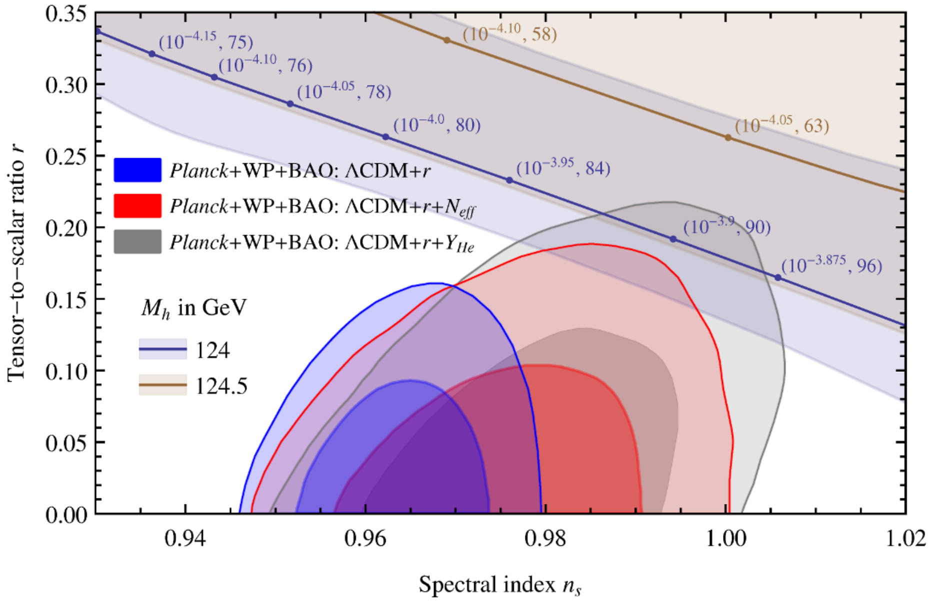

Figure 4. Predictions for the spectral index n_s and the tensor-to-scalar ratio r for Higgs ξ-inflation with prescription II and $\lambda_{\text{eff}}^{\text{min}} \sim 10^{-4}$. The solid blue (brown) curve gives the results for a Higgs mass $M_h = 124\,\text{GeV}$ ($124.5\,\text{GeV}$) while the lower and upper shaded regions correspond to a shift in $\alpha_s(M_Z) = 0.1184$ of up to $\pm 2\sigma$ (± 0.0014), respectively. The marked points along the solid curves indicate values of $(\lambda_{\text{eff}}^{\text{min}}, \xi_0)$. Results are shown with the marginalized joint 68% and 95% confidence level regions from *Planck* 2013 [7].

increase observed in [35]. Also note that the variation in n_s over the allowed range of $\lambda_{\text{eff}}^{\text{min}}$ is larger for prescription II than for prescription I. Since a deviation from the tree-level prediction of $\Delta n_s \gtrsim 0.01$ should be visible by *Planck* [62], it may therefore be possible to connect a measurement of the spectral index with the RG evolution of $\lambda_{\text{eff}}(\mu)$ near the Planck scale for prescription II. The running of the spectral index $dn_s/d\ln k$ always remains quite small, within the range $(4.5\text{–}6.4) \times 10^{-4}$.

While the results of the larger $\lambda_{\text{eff}}^{\text{min}}$ region for prescription II are qualitatively similar to those for prescription I, prescription II also allows a region of smaller $\lambda_{\text{eff}}^{\text{min}}$ and ξ_0 with distinct inflationary predictions. The existence of this region, which has not been considered in the literature before, can be understood as follows. For typical Higgs ξ-inflation with large $\lambda_{\text{eff}}^{\text{min}}$, the slow roll parameter ϵ decreases rapidly in the inflationary region (see eq. (2.9)) and the required $N = 59$ e-folds of inflation are produced quickly (see eq. (2.12)). For smaller $\lambda_{\text{eff}}^{\text{min}}$ and ξ_0, however, there is a region of parameter space in which the running of $\lambda_{\text{eff}}(\mu)$ causes ϵ to increase before the $N = 59$ e-folds are reached. The inflationary observables are then computed at a field value h_0 in a qualitatively different region of parameter space with larger ϵ, leading to distinct predictions.

Figure 4 gives the numerical results for ξ_0 and the inflationary predictions for the spectral index n_s and the tensor-to-scalar ratio r in this region. The results are shown together with the most recent constraints from *Planck* [7]. Since many well-motivated models with Higgs ξ-inflation contain additional degrees of freedom that can contribute to

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the effective number of neutrino species $N_{\rm eff}$ (e.g. the νMSM [63, 64] with 3 light sterile neutrinos), it is most appropriate to compare the results with the ΛCDM+r+$N_{\rm eff}$ data. It can be seen that, for $\lambda_{\rm eff}^{\rm min} \sim 10^{-3.9}$ and $M_h \simeq 124\,{\rm GeV}$, there is a region of Higgs ξ-inflation that is consistent with *Planck* at the 2–3σ level.[19] This region, though marginally disfavoured, is important for two reasons. First, unlike for the larger $\lambda_{\rm eff}^{\rm min}$ region, the tensor-to-scalar ratio $r \gtrsim 0.15$ in this region is quite large and would be visible by *Planck* [62]. It is therefore possible that the tensor modes from Higgs ξ-inflation could be detected in the *Planck* polarization data.[20] Second, the non-minimal coupling $\xi_0 \sim 90$ required in this region is about an order of magnitude smaller than previously considered in the literature. Although still not small enough to address the problem of perturbative unitarity violation occurring below the inflationary scale, it provides the lower limit on ξ_0 that is acceptable for Higgs ξ-inflation. Smaller non-minimal couplings, including $\xi_0 \sim 1$, seem generally unattainable because they require $\lambda_{\rm eff}^{\rm min} \lesssim 10^{-6}$ to give the correct CMB normalization, which ultimately causes the effective potential to develop a second minimum before the inflationary scale.

5 Conclusions

Higgs ξ-inflation is an attractive model of inflation since it does not require scalar degrees of freedom in addition to those of the SM. For a large non-minimal coupling ξ, however, the violation of perturbative unitarity at the scale $M_{\rm Pl}/\xi \ll M_{\rm Pl}$ threatens the self-consistency of the model in the inflationary region. In this paper we have investigated the possibility that a Higgs mass $M_h \simeq 125$–$126\,{\rm GeV}$ — a mass for which the effective Higgs quartic coupling $\lambda_{\rm eff}(\mu)$ runs to very small values near the Planck scale — may significantly reduce the size of ξ required for inflation and address the perturbative unitarity violation problem. This possibility, like the Higgs ξ-inflation scenario in general, requires a top quark mass $M_t \sim 171\,{\rm GeV}$, about 2σ below its central value.

To investigate this possibility we have updated the two-loop analysis of Higgs ξ-inflation to include the three-loop SM beta functions for the gauge couplings as well as the leading three-loop terms for the RG evolution of λ, the top Yukawa coupling y_t, and the Higgs anomalous dimension γ. We have also included, for the first time, a complete two-loop insertion of suppression factors for the physical Higgs loops in the RG equations. The two-loop SM effective potential with particle masses modified appropriately for Higgs ξ-inflation has been used to match the level of the RG equations.

We have found that successful inflation in the region $\lambda_{\rm eff}(\mu) \ll 1$ requires smaller ξ than previously considered in the literature, but even with a fine-tuning of parameters to give

[19] Recall that even with the Higgs mass measurement of $M_h = 125.7 \pm 0.4$ [52], using $M_h \simeq 124\,{\rm GeV}$ in the RG evolution is still quite reasonable due to the theoretical uncertainty in determining λ at the electroweak scale.

[20] The running of the spectral index in this region is also much larger (and negative) than typically found in Higgs ξ-inflation: $dn_s/d\ln k \sim -0.002$ and -0.008 for $M_h = 124$ and $124.5\,{\rm GeV}$, respectively. A large running of the spectral index relaxes the constraints on r from *Planck* [7] and could open up even more of the small $\lambda_{\rm eff}^{\rm min}$ region for detection.

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arbitrarily small $\lambda_{\rm eff}^{\rm min}$ it is not possible to achieve $\xi \sim 1$ and prevent the violation of perturbative unitarity below the inflationary scale. Specifically, we have found that the Einstein frame renormalization prescription (prescription I) allows a non-minimal coupling as small as $\xi \sim 400$ for $\lambda_{\rm eff}^{\rm min} \sim 10^{-4.4}$ without the potential developing a second minimum and hence spoiling inflation. The predictions for the spectral index n_s and the tensor-to-scalar ratio r remain close to their tree-level values in this case and are within the 1σ allowed region from CMB measurements. For the Jordan frame renormalization prescription (prescription II), there are two distinct regions of $\lambda_{\rm eff}^{\rm min}$ that can lead to successful inflation. The larger $\lambda_{\rm eff}^{\rm min}$ region behaves similarly to prescription I and allows a non-minimal coupling as small as $\xi \sim 2000$ without the potential developing a second minimum. The smaller $\lambda_{\rm eff}^{\rm min}$ region, which has not been considered in the literature before, requires $\xi \sim 90$ and predicts an observable tensor-to-scalar ratio $r \gtrsim 0.15$ for $\lambda_{\rm eff}^{\rm min} \sim 10^{-3.9}$. Smaller non-minimal couplings, including $\xi \sim 1$, seem generally unattainable since they require $\lambda_{\rm eff}^{\rm min} \lesssim 10^{-6}$ to give the correct CMB normalization, which ultimately causes the effective potential to develop a second minimum before the inflationary scale.

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A Renormalization group equations for Higgs ξ-inflation

In this appendix we list the (gauge-independent) RG equations for the couplings λ, y_t, g', g, g_s, and ξ in the $\overline{\rm MS}$ scheme that are used in our analysis of Higgs ξ-inflation. For each coupling we write $dx/dt = \beta_x$, where $t = \ln(\mu/\mu_0)$. The anomalous dimension of the Higgs field γ in the Landau gauge, for use in (3.14), is also given. As described in section 3.1, the RG equations contain one suppression factor $s = s(h)$ for each off-shell physical Higgs propagator.[21] Note that the RG equations for the SM can be recovered by setting $s = 1$.

The two-loop RG equations for λ, y_t, g', g, g_s, and ξ are as follows. For the Higgs quartic coupling we have

$$\beta_\lambda = \frac{1}{(4\pi)^2}\left[\left(6+18s^2\right)\lambda^2 - 6y_t^4 + \frac{3}{8}\left(2g^4 + \left(g^2+g'^2\right)^2\right) + \left(-9g^2 - 3g'^2 + 12y_t^2\right)\lambda\right]$$
$$+ \frac{1}{(4\pi)^4}\left[\frac{1}{48}\left(\left(912+3s\right)g^6 - \left(290-s\right)g^4g'^2 - \left(560-s\right)g^2g'^4 - \left(380-s\right)g'^6\right)\right.$$
$$+ \left(38-8s\right)y_t^6 - y_t^4\left(\frac{8}{3}g'^2 + 32g_s^2 + \left(12 - 117s + 108s^2\right)\lambda\right)$$

[21] The suppression factor $s(h) = s(h(\mu))$ can be written in terms of μ by inverting $\mu = h/\Omega$ or $\mu = h$ for prescriptions I or II, respectively (see section 3.2).

$$
\begin{aligned}
&+\lambda\left(-\frac{1}{8}\left(181+54s-162s^2\right)g^4+\frac{1}{4}\left(3-18s+54s^2\right)g^2g'^2\right.\\
&\left.+\frac{1}{24}\left(90+377s+162s^2\right)g'^4+\left(27+54s+27s^2\right)g^2\lambda+\left(9+18s+9s^2\right)g'^2\lambda\right)\\
&-\left(48+288s-324s^2+624s^3-324s^4\right)\lambda^2\ +y_t^2\left(-\frac{9}{4}g^4+\frac{21}{2}g^2g'^2-\frac{19}{4}g'^4\right.\\
&\left.\left.\left.+\lambda\left(\frac{45}{2}g^2+\frac{85}{6}g'^2+80g_s^2-\left(36+108s^2\right)\lambda\right)\right)\right)\right]. \qquad \text{(A.1)}
\end{aligned}
$$

For the top quark Yukawa coupling we have

$$
\begin{aligned}
\beta_{y_t}=&\frac{y_t}{(4\pi)^2}\left[-\frac{9}{4}g^2-\frac{17}{12}g'^2-8g_s^2+\left(\frac{23}{6}+\frac{2}{3}s\right)y_t^2\right]\\
&+\frac{y_t}{(4\pi)^4}\left[-\frac{23}{4}g^4-\frac{3}{4}g^2g'^2+\frac{1187}{216}g'^4+9g^2g_s^2+\frac{19}{9}g'^2g_s^2-108g_s^4\right. \qquad \text{(A.2)}\\
&\qquad\left.+\left(\frac{225}{16}g^2+\frac{131}{16}g'^2+36g_s^2\right)sy_t^2+6\left(-2s^2y_t^4-2s^3y_t^2\lambda+s^2\lambda^2\right)\right].
\end{aligned}
$$

For the gauge couplings g', g, and g_s we have

$$
\beta_{g'}=\frac{g'^3}{(4\pi)^2}\left[\frac{81+s}{12}\right]+\frac{g'^3}{(4\pi)^4}\left[\frac{199}{18}g'^2+\frac{9}{2}g^2+\frac{44}{3}g_s^2-\frac{17}{6}sy_t^2\right], \qquad \text{(A.3)}
$$

$$
\beta_{g}=\frac{g^3}{(4\pi)^2}\left[-\frac{39-s}{12}\right]+\frac{g^3}{(4\pi)^4}\left[\frac{3}{2}g'^2+\frac{35}{6}g^2+12g_s^2-\frac{3}{2}sy_t^2\right], \qquad \text{(A.4)}
$$

$$
\beta_{g_s}=\frac{g_s^3}{(4\pi)^2}\left[-7\right]+\frac{g_s^3}{(4\pi)^4}\left[\frac{11}{6}g'^2+\frac{9}{2}g^2-26g_s^2-2sy_t^2\right]. \qquad \text{(A.5)}
$$

And for the non-minimal coupling ξ we have

$$
\begin{aligned}
\beta_\xi=&\frac{1}{(4\pi)^2}\left(\xi+\frac{1}{6}\right)\left[-\frac{3}{2}g'^2-\frac{9}{2}g^2+6y_t^2+(6+6s)\lambda\right]\\
&+\frac{1}{(4\pi)^4}\left(\xi+\frac{1}{6}\right)\left[\left(-\frac{199}{16}+\frac{27}{8}s\right)g^4+\left(-\frac{3}{8}+\frac{9}{4}s\right)g^2g'^2+\left(\frac{3}{2}+\frac{485}{48}s\right)g'^4\right.\\
&+\left(\frac{45}{4}g^2+\frac{85}{12}g'^2+40g_s^2\right)y_t^2+\left(18-\frac{63}{2}s\right)y_t^4+\left(36g^2+12g'^2-36y_t^2\right)(1+s)\lambda\\
&\left.+\left(-108+126s-144s^2+66s^3\right)\lambda^2\right]. \qquad \text{(A.6)}
\end{aligned}
$$

In addition, the Higgs anomalous dimension $\gamma=-d\ln h/dt$ is given by

$$
\begin{aligned}
\gamma=&-\frac{1}{(4\pi)^2}\left[\frac{9}{4}g^2+\frac{3}{4}g'^2-3y_t^2\right]\\
&-\frac{1}{(4\pi)^4}\left[\frac{271}{32}g^4-\frac{9}{16}g^2g'^2-\frac{431}{96}sg'^4-\frac{5}{2}\left(\frac{9}{4}g^2+\frac{17}{12}g'^2+8g_s^2\right)y_t^2+\frac{27}{4}sy_t^4-6s^3\lambda^2\right].
\end{aligned} \qquad \text{(A.7)}
$$

The RG equations (A.1)–(A.7) can easily be extended to include (i) the complete three-loop expressions for the gauge coupling beta functions [44] and (ii) the leading three-loop corrections to β_λ, β_{y_t}, and γ [45]. These improvements can be made by adding the following terms to the beta functions:

$$
\begin{aligned}
\Delta\beta_\lambda = \frac{1}{(4\pi)^6}\Big[&(7176+4032\zeta_3)\,\lambda^4 + 1746 y_t^2\lambda^3 + (1719+1512\zeta_3)\,y_t^4\lambda^2 \\
&+\left(\frac{117}{4}-396\zeta_3\right)y_t^6\lambda - \left(\frac{1599}{4}+72\zeta_3\right)y_t^8 + (-2448+2304\zeta_3)\,g_s^2y_t^2\lambda^2 \\
&+(1790-2592\zeta_3)\,g_s^2y_t^4\lambda\,(-76+480\zeta_3)\,g_s^2y_t^6 + \left(\frac{2488}{3}-96\zeta_3\right)g_s^4y_t^2\lambda \\
&+\left(-\frac{532}{3}+64\zeta_3\right)g_s^4y_t^4\Big], \qquad\qquad (A.8)
\end{aligned}
$$

$$
\begin{aligned}
\Delta\beta_{y_t} = \frac{y_t}{(4\pi)^6}\Big[&-36\lambda^3 + \frac{15}{4}y_t^2\lambda^2 + 198 y_t^4\lambda + \left(\frac{339}{8}+\frac{27}{2}\zeta_3\right)y_t^6 + 16 g_s^2y_t^2\lambda \\
&-157 g_s^2y_t^4 + \left(\frac{3827}{6}-228\zeta_3\right)g_s^4y_t^2 + \left(-\frac{4166}{3}+640\zeta_3\right)g_s^6\Big], \qquad (A.9)
\end{aligned}
$$

$$
\begin{aligned}
\Delta\beta_{g'} = \frac{g'^3}{(4\pi)^6}\Big[&\frac{1315}{64}g^4 + \frac{205}{96}g^2g'^2 - \frac{388613}{5184}g'^4 - g^2g_s^2 - \frac{137}{27}g'^2g_s^2 + 99g_s^4 \\
&- y_t^2\left(\frac{785}{32}g^2 + \frac{2827}{288}g'^2 + \frac{29}{3}g_s^2\right) + \frac{315}{16}y_t^4 + \lambda\left(\frac{3}{2}g^2+\frac{3}{2}g'^2-3\lambda\right)\Big], \\
&\qquad\qquad (A.10)
\end{aligned}
$$

$$
\begin{aligned}
\Delta\beta_{g} = \frac{g^3}{(4\pi)^6}\Big[&\frac{324953}{1728}g^4 + \frac{291}{32}g^2g'^2 - \frac{5597}{576}g'^4 + 39g^2g_s^2 - \frac{1}{3}g'^2g_s^2 + 81g_s^4 \\
&- y_t^2\left(\frac{729}{32}g^2 + \frac{593}{96}g'^2 + 7g_s^2\right) + \frac{147}{16}y_t^4 + \lambda\left(\frac{3}{2}g^2+\frac{1}{2}g'^2-3\lambda\right)\Big], \\
&\qquad\qquad (A.11)
\end{aligned}
$$

$$
\begin{aligned}
\Delta\beta_{g_s} = \frac{g_s^3}{(4\pi)^6}\Big[&\frac{109}{8}g^4 - \frac{1}{8}g^2g'^2 - \frac{2615}{216}g'^4 + 21g^2g_s^2 + \frac{77}{9}g'^2g_s^2 + \frac{65}{2}g_s^4 \\
&- y_t^2\left(\frac{93}{8}g^2 + \frac{101}{24}g'^2 + 40g_s^2\right) + 15y_t^4\Big], \qquad (A.12)
\end{aligned}
$$

$$
\begin{aligned}
\Delta\gamma = -\frac{1}{(4\pi)^6}\Big[&36\lambda^3 + \frac{135}{2}y_t^2\lambda^2 - 45y_t^4\lambda - \left(\frac{789}{16}+9\zeta_3\right)y_t^6 \\
&- \left(\frac{15}{2}-72\zeta_3\right)g_s^2y_t^4 - \left(\frac{622}{3}-24\zeta_3\right)g_s^4y_t^2\Big], \qquad (A.13)
\end{aligned}
$$

where $\zeta_3 \equiv \zeta(3) \simeq 1.202$.

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